Estimates of the Natural Rate of Interest for Russia: Is ‘Navigating by the Stars’ Useful?¹

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We estimate the natural rate of interest for Russia in the short term and long term using three definitions of the rate and discuss the possible implications of the results for monetary policy. To start with, we consider partial equilibrium (under no-arbitrage condition), which is presented in the papers on estimating the natural rate. The estimates turn out to be extremely sensitive to assumptions about model parameters. The estimates based on the uncovered interest rate parity, though dependent only on observable (market) variables, impose an additional strong assumption of the path of the future equilibrium exchange rate. We supplement these calculations with calculations in panel data (for long-term equilibrium) and using semi-structural methods (for current equilibrium). To get estimates according to the strict definition of the natural rate we estimate a real business cycle model of the resource-based economy with investments using Russian data. All the estimates are highly uncertain. Taking into account the latter, the central bank should use robust monetary policy rules and avoid communicating the natural rate at least until there has been a sufficient history of business cycles in Russia.

Keywords: *natural rate of interest, real business cycle model, potential growth, uncovered interest rate parity, commodity-exporting small open economy*

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1. Introduction

Since Woodford (2000), the equilibrium (neutral) interest rate has been an integral element of all modern macroeconomic models used to make monetary policy decisions.² Despite its theoretical significance, central banks do not always publicly refer to this concept in their policies.³

In keeping with the literature on this topic, we consider three definitions of the equilibrium interest rate. For each definition, we build corresponding models, estimate the level of the equilibrium interest rate in Russia and analyse the degree of their uncertainty.

First definition. The equilibrium interest rate is the level that will be established in the economy in the long term – that is, after the effects of all shocks cease.

Second definition. The equilibrium interest rate is the level that the economy would have with flexible prices – that is, the level that corresponds to output which is at the potential level at every moment of time.⁴ In the real economy, this level is non-observable because of price rigidities.

Third definition. The equilibrium interest rate is the interest rate obtained using a semi-structural model with filters and that characterises a certain non-cyclic level of rates. This definition is the vaguest; therefore, here, we will define the equilibrium rate as the rate obtained using the Laubach and Williams model (2003), which is widely used to filter the ‘equilibrium’ rate.

The first definition places emphasis on a long-term period where the economy is in long-term equilibrium⁵ with inflation that equals the central bank’s target and GDP coincides with the potential level. Such an estimate will serve as a benchmark for interest rates in the economy over a long period. The second definition considers the equilibrium rate in the current period when the rate is influenced by various real shocks and may change immediately after the realisation of these shocks. The third definition also considers the short-term aspect.⁶

² The term equilibrium (or neutral or natural – we use these terms interchangeably) interest rate generally refers to the real equilibrium interest rate. The ‘reality’ of the interest rate means the interest rate exists even in an economy without money, simply as the price of real resources on credit. It is the price of resources that can be transferred from the next period to the current one. In practice, knowing the level of the real rate, we can get an idea of the nominal rate. After that, nominal variables (inflation expectations or the inflation target of the central bank) are simply added to the real equilibrium rate to understand its level in practice: $i^* \stackrel{\text{def}}{=} r^* + \pi^e$, where i^* is the nominal natural/equilibrium interest rate, r^* is the real natural/equilibrium interest rate and π^e is the inflation expectations (or inflation target) over the interest rate term horizon.

³ Note the difference in the practices of the Federal Reserve System (FRS) and the European Central Bank (ECB): very extensive use by the FRS (Yellen, 2015), and complete disregard in statements of Mario Draghi, President of the ECB (Draghi, 2016). Works on analysing the role of this concept in monetary policy include, for example, Amato (2005) and Carlstrom and Charles (2016).

⁴ This definition is closely related to the definition of potential output and corresponds to the so-called ‘natural’ potential output (Vetlov et al., 2011).

⁵ Hereafter, we will use the term long-term equilibrium to mean a deterministic condition of equilibrium for the Dynamic Stochastic General Equilibrium (DSGE) model. We will consider only the linear model without regard to higher-order effects.

⁶ The difference between the definitions is described, for example, by Amato (2005), and even mentioned in the FRS’ public statements (see Yellen, 2015).

In addition to these three definitions, we consider three definitions of the equilibrium rate obtained in a partial equilibrium, from certain optimality conditions of the solutions of economic agents (consumers, investors). In this case, the interest rate will be an equilibrium rate only when it meets the given condition and may, therefore, be inconsistent with the above definitions of the equilibrium interest rate. However, taking partial equilibria into consideration, first, is in keeping with the tradition in applied research on equilibrium rates, and, second, makes it possible to get a first insight into their level and determine the reliability of such estimates (by their sensitivity to a change in parameters).

As a result, we obtain estimates of the equilibrium interest rate for Russia within the framework of the basic existing main approaches – that is:

Three estimates of the neutral rate based on the optimality conditions which are part of the definition of the equilibrium rate in the overall long-term equilibrium – the first definition:

- a) the choice households make between current and future consumption (savings);
- b) the choice of the form of savings: in the form of physical or financial assets, and the respective yield parity between physical capital and financial assets;
- c) the choice of the form of savings in financial assets: national or foreign assets – parity of interest rates.

Estimates of the neutral rate within the framework of general equilibrium: DSGE model with flexible prices (Real Business-Cycle, RBC model) – these estimates are in line with the second definition.

Estimates of the neutral rate using semi-structural econometric tools:

- a) cross-border panel regressions;
- b) replication of the Laubach and Williams model (2003), initially built for a non-cyclic trajectory of the interest rate in the US economy, using Russian data. The latter method makes it possible to obtain estimates that are in line with the third definition.

Proceeding from our understanding and that presented in the literature of the rationale for the application of the 'equilibrium interest rate' in the monetary policy of central banks and economic agents external to the central bank (the financial market, business, households and the public sector), we analyse to what extent estimates obtained for Russia can be useful for the central bank.⁷ Our results, when applied to the estimate of the neutral rate for Russia with the use of different

⁷ Jerome H. Powell, Chairman of the US Federal Reserve, figuratively compared monetary policy to navigation by the stars. The stars represent the many equilibrium parameters in the economy, such as the neutral interest rate, the potential GDP and the non-accelerating inflation rate of employment (NAIRU). 'Navigating by the stars can sound straightforward. Guiding policy by the stars in practice, however, has been quite challenging of late because our best assessments of the location of the stars have been changing significantly' (Powell, 2018), which raises the question of whether such navigation is expedient.

empirical approaches, show that the definitions of the 'equilibrium interest rate' rather strongly depend on the model used, but also a very high uncertainty in estimates obtained when using particular models in practice in addition to the overall uncertainty caused by the large number of possible models.

The goal of our work is, first, to empirically demonstrate the high uncertainty of estimates of the neutral rate in the context of each standalone approach and all generally accepted approaches overall. Second, we strive to demonstrate that the potential search for the robust estimates of the equilibrium interest rate by intersecting the confidence intervals of the estimates pursuant to various approaches will not achieve the more practical objective of defining estimates of the neutral interest rate on which the central bank could confidently and explicitly rely within the framework of its communication policy.

The range of estimates for the real equilibrium interest rate (in terms of the government bond rate or the money market rate with a one-year maturity) appeared to be as follows:

1. Within the framework of partial equilibriums – optimality conditions:
 - a) The estimate based on a connection between the rate and the expected trajectory of income (potential output) affecting the intertemporal decision on consumption, given 95% confidence intervals for parameters and a range of estimates of potential growth, turned out to be the interval of [0.3; 83.6]% per annum.
 - b) The estimate based on the parity of the yield of financial and real assets turned out to be the interval of [5.5; 12.5]% per annum due to uncertainty in the estimate of the marginal product of capital in Russia.
 - c) The estimate for a short-term period based on the parity of national and international interest rates is the interval of [2.5; 3.0]%, and for a long-term period it is [2.7; 3.5]%. However, the range of estimates is extremely sensitive to assumptions about the equilibrium change of the real exchange rate and the risk premium.
2. Estimates obtained using the RBC model indicate a point estimate in 2016 of about 15% per annum. But such estimates, due to their nature, assume large confidence intervals (± 10 percentage points, p.p.), not only due to the fact that estimates of the parameters themselves have an uncertainty but also due to an additional reason related to the uncertainty arising when applying Kalman filter for the estimate of non-observable values.
3. Estimates in semi-structural models:
 - a) The 95% confidence interval of the estimates in the panel regression is very wide at [-10; +12]%.
 - b) The point estimate in the framework of Laubach and Williams model turned out to be at about 0.5% per annum. Its 90% confidence interval [-1.5; 2.5]% has little practical value for decision-making in the area of monetary policy, especially when monetary conditions are already close to neutral.

As follows from our calculations, high uncertainty in estimates may be caused not only by the uncertainty in the estimates of other non-observable variables and the choice of certain prerequisites within the framework of a particular approach; it may be of a different nature as well.⁸ Along with that, even if we attempt to cross the medians of confidence intervals from each estimated model, it is unlikely to help us understand how to quantify a more or less ‘stable’ neutral rate.

Consequently, we draw the conclusion that estimates of the equilibrium interest rate are of little practical use to the regulator.⁹ These estimates remain important for central banks as a piece of scientific knowledge that the central bank can use under certain circumstances and with appropriate reservations as a mean of enhancing its openness; for building trust in the current monetary policy, especially during the first years of inflation targeting; and to explain the theoretical background behind its decision-making. Thus, mentioning the equilibrium interest rate in rhetoric formally demonstrates to an external audience (especially economists) that the regulator is not ignoring this concept, although it understands its practical vulnerability. However, frequent use of estimates of such a non-observable variable in communications can be extremely dangerous for central banks, especially when monetary conditions in the economy in the course of normalisation of the monetary policy are near to a neutral level. Possible alternatives involve using only statistically observed indicators, such as GDP, inflation, PMI indices, industrial production capacity utilisation, unemployment indicators, lending growth, etc., for policy purposes and in communications.

The first fundamental problem of using the ‘equilibrium interest rate’ in practice is its basic non-observability. At any point of time, the actually observed interest rate (nominal or real) is the point equilibrium rate for the particular short-term economic equilibrium. However, the term ‘equilibrium interest rate’ implies something different – a certain interest rate that will be reached under specific conditions; in real life, however, we can never be sure that these conditions are met. That is why the equilibrium interest rate does not exist outside a theoretical model that sets conditions that make it possible to observe the rate.

Here, the first challenge arises as there are various concepts of the neutral/equilibrium rate. For example, in its standard definition, the neutral rate is established when output and inflation are in equilibrium (Gali, 2008, p. 49; Woodford, 2003; Barsky et al., 2014). However, as noted by Juselius et al. (2017) and Borio et al. (2019), this is insufficient if a financial cycle is taken into account

⁸ For example, when estimates are made within the framework of a semi-structural approach with the use of cross-border panel data, wide confidence intervals are caused by great practical difficulties in fully accounting for economies’ structures, etc.

⁹ We are not the only ones who have come to such a conclusion; see Beyer and Wieland (2019) and Taylor and Wieland (2016). It should be noted that it can be even harder to measure the nominal equilibrium rate compared to the real one due to the fact that the level of inflation expectations cannot be observed in practice unless such expectations are assumed to be anchored to the central bank’s target. Since inflation expectations are not observable, it is equally problematic even to measure the actual real interest rate in the economy.

in addition to a business cycle. When considering the consequences for a financial cycle for financial stability, an equilibrium rate is likely to be higher compared to models that take into account only a business cycle.

The second challenge, even in the context of a single concept (for example, Neo-Keynesian), is that equilibrium rates can take different values depending on which model is used:

- an open or a closed economy (Hledik and Vlcek, 2018; Wynne and Zhang, 2018);
- an economy with financial rigidities or an economy without rigidities (Eggertsson and Krugman, 2012; Caballero and Krishnamurthy, 2002).

Even such a factor as the trust of economic agents in the central bank can turn out to be crucial to the level of the equilibrium rate, as shown by Tristani (2009).

It follows that overcoming the fundamental problem of using the equilibrium rate concept in practice requires a clear definition of this rate for policy purposes as well as explanation of this definition to the public and understanding of the limits of its application.

The second challenge of the practical application of this concept is the difficulty of estimating the neutral rate. As Robert Kaplan, President of the Federal Reserve Bank of Dallas, mentioned in his essay (Kaplan, 2018), estimating an interest rate that would be consistent with neutral monetary conditions is 'more of an art than a science and involves observing and analysing a wide variety of factors'.

The estimation is difficult because in real life a researcher or a policy maker cannot be sure if the conditions set in the model for determining the equilibrium rate are met. Some conditions, such as price flexibility requirements, are not met with a high probability. In other words, in reality, we do not observe an economy with flexible prices and obtain statistical data for estimating model parameters from the actual economy with rigid prices. Moreover, there are no grounds for believing that model parameterisation based on statistical data on flexible prices economy will be the same as parameterisation based on data about the actually-observable economy. In other words, there is a high probability that the parameters of macroeconomic models used to make equilibrium rate estimates are biased, which also entails the bias in estimates of the equilibrium rate.

Another challenge associated with estimation is the high uncertainty of the resulting estimates in practice (Beyer and Wieland, 2019). We emphasise this fact in this paper by means of calculations based on data for the Russian economy. The high uncertainty is also associated with the estimates of other non-observable indicators, such as the potential GDP or the NAIRU. We believe that in such circumstances real rate estimates need to be thoroughly verified for robustness on a continuous basis.

The source of the uncertainty in the estimates in the context of each approach is the uncertainty of model parameters or the inaccurate measurement of factors of the equilibrium interest rate. This directly follows from the three no-arbitrage conditions:

1. Consumer choice equilibrium (Euler equation) – no arbitrage between the desire of households to save and consume.

The main equation for estimation (Gali, 2008, p. 49) is:

$$r^* = \sigma g^* - \ln \beta, \quad (1)$$

where r^* is the equilibrium real interest rate, g^* is the growth rate of potential GDP per capita (income or consumption), σ is the inverse elasticity of intertemporal substitution and β is the subjective rate of intertemporal preferences (discount rate).

As one may see, this method requires estimates of three non-observable values (potential growth, discount rate and intertemporal substitution rate) to estimate one non-observable value. This makes us sceptical as to the practical utility of this approach.

2. No arbitrage between investments in financial assets (at some interest rate) and in physical capital. This condition is described by the following equation (Hall and Jorgenson, 1967; Creedy and Gemmell, 2015):¹⁰

$$r + \delta - (\Delta P_{capital} - \Delta P_{GDP}) = MPK, \quad (2)$$

where MPK is the marginal product of physical capital (in terms of real GDP), $r + \delta - (\Delta P_{capital} - \Delta P_{GDP})$ is the user cost of capital in real terms, $(\Delta P_{capital} - \Delta P_{GDP})$ is the change in the relative price of physical capital in terms of consumer product or the GDP deflator and δ is the capital depreciation rate for the period. Equilibrium/long-term values of the respective variables yield the estimate of r^* .

3. No arbitrage between investments in domestic and foreign financial assets, or uncovered interest rate parity.

$$r = r^f + \Delta E[p_{t+1}] + risk_premia, \quad (3)$$

where r^f is the interest rate in foreign currency used for arbitrage operations, $\Delta E[p_{t+1}]$ is the expected weakening of the real exchange rate p and $risk_premia$ is the risk premium on investments in national assets. Equilibrium/long-term values of the respective variables result in the estimate r^* .

The possibility of estimating the equilibrium rate for the short-term equilibrium (up to two years) from this correlation is, as we show below, related to the possibility of fulfilling a rather specific condition, namely, to the ‘dilemma, not trilemma’ of monetary policy (see Rey, 2018). Under Robert Mandell’s standard

¹⁰ Equation (2) is sometimes adjusted, for example, for taxes. In particular, marginal corporate tax is deducted from marginal product. However, it is crucial in this case to make an adjustment for taxes and income from financial assets—that is, there must be a difference between the interest rates before and after taxation. Then, we show that adjustments for taxation—if one takes the difference between interest income tax rates and corporate tax rates in Russia into consideration—have almost no effect on equilibrium interest rate estimates resulting from the condition of equal returns on physical and financial assets. Therefore, for simplification purposes, we omit taxes in equation (2) in calculations for Russia.

monetary policy trilemma, the adjustment of the exchange rate ensures that the internal interest rate is independent from external factors (the world interest rate and, in part, the country risk premium). This makes it impossible to use the parity to estimate the internal rate when external variables change in the short-term equilibrium. The exchange rate is adjusted so that exogenous changes in the right-hand side of the equation are completely negated, and as a result, the left-hand side – that is, the equilibrium rate – is redundant. As for long-term equilibrium (beyond the monetary policy horizon), in order to apply the equation (3), there is a need for additional estimates of the average level of risk premium over a long period and the existing long-term trend of real exchange rate changes.

Another challenge in using the interest rate parity is related to the fact that it is based on information about financial markets, which, in particular, takes into consideration what the market thinks about the equilibrium interest rate, including based on information received from the regulator. A situation may arise where equilibrium rate estimates made using this method will simply reproduce the equilibrium rate communicated by the central bank; they will not contain any new information for the regulator.

The equations describing the given conditions are then estimated econometrically, or their structural parameters are simply calibrated (selected based on other considerations). The ‘equilibrium’ interest rates obtained in such a way may not necessarily coincide due to imprecise estimates of parameters or data. Their consistency can, for example, be ensured by building a general equilibrium model that provides insight into the mechanism of the convergence of interest rates from different conditions and which equilibrium they all finally converge towards, which of the three equations is the primary one in this context.¹¹ It is important that the estimates considered separately need not necessarily correspond to the definitions of the equilibrium rate mentioned above; one should remember this when encountering such ‘equilibrium’ rate estimates in the literature. As we show in the paper, only the first condition taken separately – namely, the relation between the interest rate and potential GDP growth rates – meets one of the definitions (namely, the first definition) of the equilibrium interest rate.¹² Therefore, to estimate the equilibrium interest rate over the long term, we focus on this correlation first, and, in addition to calibrating it, we estimate the panel regression of the relation between the interest rates and per-capita GDP growth rates (see, for instance, He et al., 2015; Hamilton et al., 2015).

As a rule, to estimate the interest rate in the short term, all of the three conditions are considered together – in general equilibrium in RBC models (see Mendoza, 1991; Guo and Janko, 2009), as in the short term transitional dynamics is crucial, and there is no way to determine it otherwise.

¹¹ We will show below that in standard general equilibrium models of a small, open (resource-based) economy the long-term equilibrium rate is determined based on the first condition, which links the interest rate level and the rates of potential GDP growth (or, to be more precise, consumption growth rates).

¹² The rates from the other two conditions must converge towards the rate set by the first condition.

That is why, to assess the equilibrium rate in the short term, we built a general equilibrium model for Russia without price rigidities (with flexible prices), or an RBC model. In this model, the economy has the characteristic features of the Russian economy – a small open economy exporting resources. For our purposes, we considered a three-sector structure of the economy in general equilibrium (for the energy resource extraction sector, the domestic production sector, and the overseas production sector) and took capital accumulation into consideration (firms optimally determine capital stock and make investments). We estimated the model using Russian statistical data.¹³ In this model, we focused not only on obtaining the level of the interest rate in short-term equilibrium but also on analysing the transitional dynamics of the interest rate from one long-term equilibrium to another.

To estimate the interest rate under the third definition, we replicated one of the most cited filters on this subject, Laubach and Williams (2003, 2015), and provide the respective estimates. These estimates reflect the estimated short-term equilibrium interest rate.

Further, the material is arranged as follows. In Section 2, we provide a summary of literature with a description of theoretical approaches to determining the equilibrium interest rate and the results of their empirical verification. In Section 3, we consider the elements of the general equilibrium model that are key to determining the interest rate – namely, the three no-arbitrage equations. In Section 4, we assess the general equilibrium model, obtain equilibrium interest rate estimates and analyse the response of the rate to changes in individual variables or parameters (oil prices, global rate, changes in the subjective discount factor). In Section 5, we provide equilibrium rate estimates in semi-structural models – that is, according to the third definition, and equilibrium rate estimates in the model based on panel data. In Section 6, we summarise the main results and draw conclusions for the Central Bank's policy.

2. Literature overview

We do not know of any published articles that estimate the equilibrium rate for Russia and provide confidence intervals of such estimates besides the preprint by Klose (2018), where the interval 'plus/minus one standard deviation of the estimate' has a range of around 20 p.p. for the annual interest rate.

For other countries, as shown by Brand et al. (2018) and Beyer and Wieland (2019), estimates of equilibrium interest rates obtained using various methods are characterised by very high uncertainty, which has crucial consequences for assessing the suitability of these estimates for monetary policy purposes.

An overview of approaches for determining the equilibrium interest rate using the example of the US economy is presented in the works of Woodford (2000),

¹³ The model with price rigidities is estimated, after which the model parameters are used for the model without price rigidities.

Amato (2005), Laubach and Williams (2015) and Pescatori and Turunen (2015). Empirical estimates of the equilibrium interest rate (including for other countries) are given in the works of Hamilton et al. (2016), Grafe et al. (2018), Brand et al. (2018) and Beyer and Wieland (2019). Real interest rate definitions and estimates are also described by Lundvall and Westermarck (2011).

The equilibrium interest rate as the rate corresponding to the long-term equilibrium of the economy (and target inflation) – that is, the first definition, is covered in the works of Fuentes and Gredig (2007) and He et al. (2015). In this case, individual equations of general equilibrium models are estimated or calibrated:

1. The Euler equation is estimated by He et al. (2015) and Fuentes and Gredig (2007). A similar concept of defining the real equilibrium rate is based on consumption-based capital asset pricing model (C-CAPM; see Giammarioli and Valla 2004). The key predictions of this model are though not confirmed in practice (Hamilton et al., 2016; Kocherlakota, 1996; Mehra and Prescott, 2003).

A similar approach involving decomposition of the consumption to wealth ratio dynamics is used by Gourinchas and Rey (2019).¹⁴

2. Estimates based on marginal capital productivity are presented by Brand et al. (2018).

In all cases, the authors are not analysing the uncertainty of estimates.

Models for estimating the interest rate in short-term equilibrium according to the second definition are presented using RBC models (or Neo-Keynesian models, but with disabled nominal rigidities); see Giammarioli and Valla (2004), Barsky et al. (2014) and Curdia et al. (2015).

Estimates based on the third definition are based mainly on the semi-structural approach described by Laubach and Williams (2003, 2015). Beyer and Wieland (2019) find that estimates obtained for the US using this popular method have very high uncertainty. The authors point out that the estimates turned out to be very sensitive to insignificant technical changes in the specifications of models or data.

Descriptive models of equilibrium between the supply of loanable funds (savings) and demand for them (investments) that examine the price of loanable funds through demand and supply factors are well represented in the literature. These models are based on individual results of various theoretical models and assess not the level of the real interest rate but rather the direction of changes in it. Most commonly in non-model descriptions (Blanchard et al., 2014; International Monetary Fund, 2014) the interest rate depends on a number of factors defining national savings, investments and investor attitudes to risk (relative demand for safe and risky assets).

Estimates of the global equilibrium rate reflect a number of widely discussed tendencies (International Monetary Fund, 2014a; Lubik and Matthes, 2015).

¹⁴ We do not use this approach for Russia because of the difficulty in obtaining similar statistical data regarding the level of assets of Russian households. A possible source of data may include data from sample surveys within the framework of the Russian Longitudinal Monitoring Survey (RLMS). However, this work is beyond the scope of this paper.

An important area in estimating the equilibrium interest rate includes papers focusing on the disequilibrium in the economy when using a standard definition of equilibrium interest rates because of the possible accumulation of financial stability risks (see Juselius et al., 2017 and Borio et al., 2019).¹⁵

3. Estimates of uncovered parity of interest rates are obtained by Fuentes and Gredig (2007) for Chile and by Perrelli and Roache (2014) for Brazil.

This approach to estimating the equilibrium rate assumes the independence of changes in its constituents (global rate, exchange rate and risk premium), which is only true under specific conditions, namely, in case of the ‘dilemma not trilemma’ of monetary policy (Rey, 2018). In case of standard trilemma of monetary policy, all external shocks are absorbed by changes in the exchange rate (adjustment of expectations of its changes) without affecting the equilibrium interest rate, thus making the focus on changes in external factors useless for estimating the equilibrium interest rate.

Starting with the empirical results of Rey (2018), there emerged a school of thought in the literature that questioned the practical feasibility of interest parity. Namely, it was claimed that whether a national central bank can conduct a monetary policy (establish an interest rate) other than that determined by external factors (global interest rate or, to a certain extent, country risk premium) depends on the feasibility of parity – that is, the ability of the exchange rate to adjust to changes in the external interest rate or country risk premium. Liquidity constraints can restrict the central bank’s ability to influence money supply in the economy, as shown by Caballero and Krishnamurthy (2002). Gourinchas (2018) formally introduces the dependence of such constraints on the exchange rate into the DSGE model; strengthening of the exchange rate weakens liquidity constraints. The stronger this dependence between the exchange rate and liquidity constraints, the stronger the dependence of the internal interest rate on the external/global interest rate. A similar dependence of internal monetary conditions on the exchange rate is described by Diamond et al. (2018).

If the above mechanisms are truly important, and there is a policy dilemma instead of trilemma, it makes sense to estimate the internal equilibrium interest rate based on external variables.

3. Equilibrium rate estimates within the framework of partial equilibriums

In this section, we describe estimates for each of the three conditions of no-arbitrage separately.

¹⁵ In our paper, we obtain estimates solely within the scope of models that do not take financial stability aspects into consideration. However, the definitions of the equilibrium rate that we use may easily be extended to cover this class of models.

3.1. Equilibrium rate and potential GDP growth

The key condition for determining the equilibrium interest rate is the optimal choice by households between current consumption and future consumption (savings), or the Euler equation:¹⁶

$$E\left[\frac{c_{t+1}}{c_t}\right]^{-\sigma} * \beta * (1 + r_t) = 1, \quad (4)$$

where c_t is the per capita consumption in the current period, c_{t+1} is per capita consumption in the next period, $E[*]$ is the expectation operator, $0 < \beta < 1$ is the intertemporal preferences rate (future discount rate) and $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution.

Equation (4) sets the demand for savings (financial assets): the higher the rate, the lower current consumption (the higher its growth for the set future consumption) and the higher the supply of resources for investments, other things being equal. The intuition for the relation between consumption growth and interest rate may be the inverse: higher consumption growth means that tomorrow's consumption (income) will be higher than current consumption. The motive of consumption smoothing (individuals' desire to have a stable consumption plan for their entire life) leads to a growth in demand for consumption today, when income is not yet sufficiently high. This causes growth in the real interest rate through increasing demand for borrowed resources to smooth consumption. As a result, a connection is established between long-term income growth (or to be more precise, consumption growth) and the real interest rate on deposit and credit operations:

$$r^* = \sigma g^* - \ln \beta, \quad (4a)$$

where g is the rate of per capita potential growth of the economy and σ is the inverse of intertemporal elasticity of substitution.

In addition to the fact that calculation of the neutral rate in the context of this approach is made within the framework of partial rather than general equilibrium,¹⁷ its main disadvantages include a lack of credit restrictions that could prevent consumption smoothing to some extent and a lack of fiscal policy effects.

We calibrate the key elasticities in equation (4a), which makes it possible to estimate the equilibrium rate over the long term for Russia (see Table 1).

We base our estimates of potential GDP growth for Russia on calculations the details of which are set forth in the Appendix 1. For simplicity, we ignore the uncertainty of such estimates, which generally does not help to reduce the overall uncertainty of equilibrium rate estimates.

¹⁶ The equation is obtained for a utility function of the type $U(C) = \frac{c^{1-\sigma} - 1}{1-\sigma}$. The problem and its solution are given by Barro and Sala-i-Martin (1999, p. 61) and Galí (2008, p. 49).

¹⁷ In general equilibrium over the long term, the equilibrium rate is determined by potential GDP growth parameters—namely, population change rates and technological progress. Other arbitrage conditions adjust to the correlation set by the Euler equation through accumulation of physical capital and net foreign assets/liabilities (risk premium change).

Table 1. Calibration of the equation parameters

Parameter	Value	Source	Comment
$\frac{1}{\sigma}$	4.167	Khvostova et al. (2014)	Intertemporal elasticity of substitution for Russia
	1	Christoffel et al. (2008)	Standard calibration
β quarterly	0.85–1.098	Khvostova et al. (2014)	95% confidence interval for the subjective discount factor for Russia
	0.99	Yogo (2006)	Standard calibration

Taking into account the confidence interval for β using to Russian microdata (Khvostova et al., 2014), as well as the substantial restriction of $\beta \leq 1$, for our estimates of per capita annual GDP growth (1.2–2.6%, obtained from the range of estimates of GDP growth offset for labour contribution), we obtain the range for the real rate of [0.3; 83.6]% per annum. The equilibrium rate estimate turns out to be very sensitive to the estimate of the subjective discount factor. Assuming the discount coefficient, which is the most often used in the literature (see Yogo, 2006), is at 0.99 per quarter (this value falls within the confidence interval for estimates based on Russian data for the quarter), we obtain a rather narrow range for the interest rate at [4.4; 4.7]% per annum.¹⁸

In general, we can conclude that the estimate of one non-observable variable based on the estimates of three non-observable variables, as in this approach, is in practice an exercise with low chances of success.¹⁹

3.2. Equilibrium rate and physical capital productivity

Let us consider the absence of arbitrage (equilibrium choice) between investments in real and financial assets. The interest rate must balance the returns on investments in financial and real assets. The no-arbitrage condition requires the following equality, which can also be interpreted as the demand for capital:

$$r + \delta - (\Delta P_{capital} - \Delta P_{GDP}) = MPK. \quad (5)$$

The expression in the left-hand side of equation (5) is the user costs of capital during the period of time set by the interest rate term. Any deviations from parity are adjusted through capital stock. Other conditions being equal, the higher the interest rate, the lower the demand for real capital.

Capital productivity growth makes investments in real assets more profitable than investments in financial assets at the same interest rate. There is a flow of resources from financial to real assets, which levels the yields from the two types of assets. In general equilibrium, other things being equal, interest rates will increase.

¹⁸ If a parameter of intertemporal elasticity of at unity (a standard value in the literature, not obtained from data on the Russian economy), the range of estimates for the equilibrium is at [5.3; 6.8] % per annum.

¹⁹ Adjusted estimates of the discount factor and intertemporal elasticity of substitution, for example, from a DSGE model or econometric estimates, can provide more accurate equilibrium rate estimates; however, for Russia, there is the problem of structural shifts and short sample length. In such models, equilibrium rate estimates will to a certain extent represent averaged historical data with a number of reservations.

From this expression follows the expression for the equilibrium interest rate:²⁰

$$r^* = \overline{MPK} + (\overline{\Delta P_{capital}} - \overline{\Delta P_{GDP}}) - \bar{\delta}, \quad (6)$$

where bar notations are the long-term (equilibrium) values of the respective parameters, and $(\overline{\Delta P_{capital}} - \overline{\Delta P_{GDP}})$ is the difference between price growth rates for the investment deflator and GDP. For further calculations of the current equilibrium interest rate, we take $\bar{\delta}$ as equal to 5% per annum. Estimating marginal capital productivity MPK poses the greatest difficulty. The simplest solution to this problem described in the literature uses the Cobb–Douglas production function (see He et al., 2015), for which marginal capital productivity is equal to

$$MPK = \alpha * \frac{Y}{K}, \quad (7)$$

where α is the share of capital in GDP and $\frac{Y}{K}$ is the capital productivity ratio (the ratio of GDP to capital stock). For Russia, $\frac{Y}{K}$ is about 0.3–0.5 on average for 2000–2014 based on various capital stock estimates that we made using data from Rosstat, World KLEMS Russia and International Monetary Fund (IMF) data.²¹ The share of capital in income is about 0.35. On average, in 2011, capital products appreciated +1% more quickly than end-products. As a result, for the interest rate, we receive the range of [5.5; 12.5]% per annum because of various capital productivity estimates in Russia.

The range of estimates for $MPK - (\overline{\Delta P_{capital}} - \overline{\Delta P_{GDP}})$ for other countries is provided by Caselli and Feyrer (2007). In Russia, the respective value is [0.083; 0.155], depending on the capital stock estimate, which turns out to be within the range for 53 countries [0.07; 0.23] but closer to its lower limit, which corresponds to developed countries.

In general, the above estimate also seems unreliable due to the complexity of estimating the marginal capital product in the economy and the capital depreciation rate, which are included in the estimate linearly.

3.3. Equilibrium rate and correlation with the global interest rate

The equilibrium of demand for domestic and foreign financial assets is described by the interest rate parity (equation (3)). A deviation of the global interest rate from

²⁰ We also estimated this ratio adjusting for taxes. We used KPMG's database (<https://home.kpmg.com/xx/en/home/services/tax/tax-tools-and-resources/tax-rates-online/corporate-tax-rates-table.html>), where the corporate tax rate for Russia is 20%. The tax rate on interest income was assumed to be equal to 35%. Accounting for tax did not affect the estimates.

²¹ The calculation was carried out based on: 1) Rosstat data on capital stock investments at 2008 prices, and the depreciation rate, which depends on the fixed capital capacity utilisation, which, with an average load over 2000–2014, was assumed to equal 5% per year (according to the Russian Economic Barometer); 2) data from World KLEMS Russia (<http://www.worldklems.net/data.htm>), in particular, the capital services indicator, which, like Rosstat data, can be used to estimate capital stock; 3) IMF data used in the International Monetary Fund paper (2014b), which we were given by the IMF upon request.

the internal interest rate, other things being equal, will entail a flow (with a certain lag) of global resources to the country or domestic resources from the country.

The ruble equilibrium interest rate should consist of the real equilibrium rate abroad²² with the same term (the US Fed's federal funds rate can be used as a proxy variable), the equilibrium sovereign risk premium and the equilibrium change in the real USD/RUB exchange rate. Let us consider each of these components separately for the short rate on the Russian money market.²³

In view of the foregoing, we would like to focus on several methods of estimating the Russian sovereign risk premium which should be taken into account in making investment decisions on buying Russian assets.

Credit Default Swap (CDS) premium-based estimates. CDS premium is a security payment for default upon government bonds in basis points over the nominal value.²⁴ To calculate the risk premium, we used two comparison bases: Russian CDS premium vs US and CDS median for a group of countries with a credit rating similar to Russia (long-term rating in foreign currency by Moody's) vs the median rating for countries with a higher credit rating (including the US). In the first instance, the risk premium will include a premium for risks specific to Russia. In our calculations we used five-year CDS premiums that, unlike one-year premiums, are a more liquid instrument, though they factor in risks over a five-year horizon and can, therefore, overstate risks and the equilibrium rate estimate over the short term.²⁵ Our econometric estimates within the time interval from January 2010 to June 2016 showed that the average five-year CDS premium for countries with a credit rating comparable to Russia (Ba1 on Moody's scale) is 190 basis points (b.p.) versus the CDS premium of countries with the highest credit rating (see Figure 1). Given that long-term credit ratings reflect long-term trends in risk assessment, the above estimate may be considered a long-term risk premium estimate – that is, + 190 b.p.

Estimates based on bonds secured against inflation (Bomfim, 2001). The estimates based on seven-year inflation-protected bonds make it possible to gain insight into the inflation expectations of financial market players (including the inflation volatility premium) and the real returns on bonds priced by market players. Comparing the real interest rate of inflation-protected bonds (Treasury Inflation-Protected Securities, TIPS) in Russia and the US allows us to get an idea

²² The portfolio of foreign currencies can also be considered in parity; this, by linearity, may be reduced to parity with one currency (US dollar), which will add changes in the exchange rates of these currencies to the US dollar to the interest parity. Therefore, it will be sufficient to consider parity with only one currency.

²³ It bears repeating that the rates estimated in this way do not, over the short term, have anything to do with the three definitions taken as a basis in this work. On the contrary, they are only an illustration of how such estimates are sometimes made in practice. These estimates do not have any special meaning in the context of short-term real equilibrium rates, but in practice they are used as their benchmark.

²⁴ It should be noted that an estimate as the ratio of the cost of a CDS to its nominal value is not a strictly mathematically correct estimate of risk premium. An accurate estimate is rather complicated and depends on a number of factors.

²⁵ Given the various liquidity premiums for long and short CDSs, the difference will be immaterial.

of the changes expected by market players in the real exchange rate and sovereign risk premium (see Figure 3).

Figure 1. CDS premium and long-term credit rating in foreign currency, b.p.

Note: along the X axis: the highest credit rating, Aaa = 21, the lowest, C = 1.

Source: Bloomberg

Figure 2. CDS premium of Russia vs US, b.p.

Figure 3. Real yield-to-maturity spread on inflation-protected bonds in Russia and the US, p.p.

Source: Bloomberg

Figure 4. Yield-to-maturity spread on Russian eurobonds and US bonds, p.p.

Estimates based on the yield spread on Russian sovereign eurobonds and US government bonds with the same maturity. This spread will also include liquidity premium (see Figure 4).

The last component for reconstructing the real equilibrium interest rate in Russia (assuming the effect of a policy dilemma) is the adjustment for the weakening of the real exchange rate expected by the market. This estimate can, for example, be obtained from analytical consensus forecasts for the nominal exchange rate and the inflation expected in a year in Russia and abroad. However,

due to the fact that the spread of such forecasts (especially in respect of the exchange rate) can be rather high, we considered it inexpedient to rely on a specific point estimate of changes in the real ruble exchange rate in our calculations and instead assumed stable real exchange rate (essentially disregarding the exchange rate variable in our calculations).

The results we obtained for the real short interest rate (money market rate) in Russia in the context of an absence of arbitrage based on the example of calculations we carried out in 2016 are presented in Table 2.

Table 2. Components of the real short equilibrium interest rate on the condition of interest rate parity (example of calculations for 2016)

	Real rate, %	Equilibrium real rate in the US, %	Risk premium estimates, p.p.	Expected weakening of the real exchange rate
Short term	2.5–3.0	0	2.5–3.0	0
Long term	2.7–3.5	1.2–1.5	1.5; 1.9; 2.0	0

As a result, considering the higher risk premiums, the equilibrium real interest rate with no arbitrage as of 2016 can be estimated at 2.7%, with the mean value of 2.5% and 3.0% in Table 2. If we take the data on five-year CDS premiums for 2019, the respective point estimate may be about 1.5% or a little higher. Also, proceeding from market expectations of inflation one year ahead (judging by Bloomberg's last consensus, analysts' medium-term inflation expectations seem to be anchored at the Bank of Russia's target level of 4%), we can conclude that the current equilibrium ruble nominal interest rate is about 6% (in our example of calculations for 2016, it would be about 9% = analysts' inflation expectations for one year ahead of 6.0%–6.5% + risk premium of 2.5%–3.0%). At the same time, we estimate the equilibrium nominal short rate over the long term at about 6.7%–7.5% (4% of inflation + 2.7% or 3.5% risk-premium).

On the one hand, the possibility of direct estimates of the interest rate parity components increases the practical value of the equilibrium interest rate estimates made using this method. On the other hand, a number of assumptions (concerning the estimates of risk premium and expected equilibrium changes in the real exchange rate) do not allow us to rely entirely on its accuracy, especially over the long term. In monetary policy communication, the equilibrium rate concept is, in our opinion, most useful to economic agents and the regulator as an estimate of the long-term equilibrium rate. The main issue is that in the case of a standard policy trilemma (as opposed to Rey's 'Dilemma' (Rey, 2018)) the estimation of the internal equilibrium interest rate through risk premium and exchange rate adjustments may not depend on the external conditions, but interest rate parity does not use any other information besides external conditions. Changes in risk premiums and exchange rates in response to the external interest rate shocks are not exogenous. This fact significantly reduces the practical value of this approach.

4. Equilibrium rate in general equilibrium

Further, we combine all three of the conditions defining the real interest rate with the help of the RBC general equilibrium model. In Appendix 2, we provide a detailed description of this model for the Russian economy. We consider a standard RBC model of a small open economy. See a similar model, but without capital, in the work of Kreptsev and Seleznev (2018).

Essential features of this model for Russia include:

- The economy's structure and structural shifts after real shocks (oil price shock).
- Capital accumulation by firms in the presence of capital adjustment costs. It is often the case that similar models used in interest rate assessments omit capital accumulation, which disables one of the conditions for determining the equilibrium interest rate; see Barsky et al. (2014).
- Optimisation conduction by households of not only their consumption but also the number of hours worked, as well as their consumption habits.
- Some parameters in the model are estimated based on statistical data of a full model that includes price rigidities and the central bank. The model is estimated using the data on real GDP (both in Russia, including expenditure GDP, and the global economy), inflation (in Russia and its trading partners) and the MIACR interest rate. Some parameters are calibrated; see Kreptsev and Seleznev (2018).

Figure 5. Money market current equilibrium real rate (MIACR) in the general equilibrium model for Russia, % per annum

The current equilibrium interest rate estimate (according to the second definition) in this model is provided in Figure 5. This estimate is very rough and has wide (about 10 p.p.) confidence intervals (not shown in the figure) that, apart from the model specification, are caused by the uncertainty in estimating parameters (a consequence of a short sampling) and the use of a Kalman filter, which does not give consistent estimates.²⁶

²⁶ The same problems arise when semi-structural models are applied.

In this model, it is more important for us to analyse the response of the equilibrium real interest rate to the following permanent shocks (changes in long-term equilibrium):

1. Demographic changes associated with growth in the share of older age groups in the demographic structure, which corresponds to lesser discounting of the future as households become more prudent. Because of the difficulties in seeking a correspondence between the ongoing demographic changes in Russia and their effect on the discount coefficient, this case begs further academic research. We assume the annual coefficient β in a formula similar to equation (4) to increase by 0.01.

2. Growth in the global interest rate by 1 p.p.

3. Decrease in the price of oil by 10%.

The results for each of the changes breaking long-term equilibrium are provided below (see also the figures in Appendix 3):

1. *An increase in the rate of intertemporal preferences (growth in the future discount coefficient by 0.01).* Such a structural shift is intended to reflect the change in the demographic structure of the population – namely, the reduced share of youth, who heavily discount their future income ('living for the moment' or impatience). The growing value of future consumption causes a reduction in the equilibrium rate in the new equilibrium by almost 1 p.p. (see Figure 6). The interest rate drops beyond its equilibrium value, decreasing by 10 p.p. After that, it grows steadily.

The equilibrium interest rate decreases because households start to actively cut down their debt (accumulate savings) for higher future consumption, which they now value more. The interest rate turns out to be 10 p.p. lower than the new long-term equilibrium. Subsequently, it will grow to a new equilibrium over the course of a year. In response to the growth in the demand for capital, the price of investment goods immediately increases, which partially offsets the lowering effect of lower rates on the cost input per unit of capital. In response, firms increase imports of investment products. Reduction of external debt entails a decrease in the risk premium, which, along with strengthening the real exchange rate, satisfies the no-arbitrage condition with a lower interest rate over the long term.

Figure 6. The response of the equilibrium rate to weaker future discounting over the long term and dynamics of transition to the equilibrium

2. *Growth of the global interest rate by 1 p.p.* does not change the long-term real rate in the model for Russia due to a decrease in the country risk premium which

compensates for growth of the global rate.²⁷ In the short term, the equilibrium rate decreases by 1.4 p.p. (drops beyond the equilibrium level) and then returns to the equilibrium level over the course of a year (see Figure 7).

The decrease in the short-term equilibrium rate is due to the rapid repayment of external debt, which turns out more costly with higher external rates. This external debt repayment reduces the risk premium significantly and thus restores the interest parity.

Figure 7. Response of the equilibrium rate to growth in the global rate by 1 p.p. in long-term equilibrium and dynamics of transition to the equilibrium

3. A 10% decrease in oil prices leaves the equilibrium rate unchanged in the long term, while over the short term it increases by 3 p.p. (see Figure 8).

Figure 8. Response of the equilibrium rate to a 10% decrease in oil prices in long-term equilibrium and dynamics of transition to the equilibrium

In general, estimates show that with standard assumptions in respect of the role of capital flows (interest rate parity and the monetary policy trilemma) external shocks (changes in oil prices or the global interest rate) do not affect the equilibrium interest rates over the long term which turn out to be dependent on the structural characteristics of the domestic economy. This suggests that when

²⁷ This result is obtained in the model because the debt level is the only country risk premium factor. If the sovereign rate turns out to be less sensitive (or insensitive) to the debt level, the interest rate inside the country will probably increase, but in our standard model of a small open economy this effect cannot be modelled due to a number of technical restrictions; for more details, see Schmitt-Grohe and Uribe (2003).

potentially communicating the long-term equilibrium rate against the backdrop of various external shocks it is necessary to indicate the strength of the frictions that may affect the economy's adaptability – that is, its correspondence to the economy described in the model. The degree of such frictions determines the sensitivity of internal long-term rates to external factors and, consequently, the ability of structural policy measures (rather than the central bank alone) to influence long-term equilibrium rates.²⁸

5. Equilibrium rate estimates using econometric methods

5.1. Estimating the long-term equilibrium rate using panel data

For the purpose of empirical estimates of the equilibrium interest rate in Russia in the long-term equilibrium, we assessed the commonly used specification (Hamilton et al., 2015; He et al., 2014) of the interest rate dependence (in our case, the money market short rate, see the description of data in Appendix 4) on the potential GDP growth rate and other factors:

$$r_{it} = c + \beta_1 g_{it} + \beta_2 s_{it} + \beta_3 KAopenness_{it} + \varepsilon_{it}, \quad (8)$$

where r_{it} is the real interest rate for country i in period t , g_{it} is the growth of potential GDP for country i in period t , s_{it} is the savings rate for country i in period t and $KAopenness_{it}$ is the economy (capital account) openness indicator, or the level of financial development.

In this specification, we essentially assess the Euler equation (equation (1)) which connects the real rate with the potential output growth rate, the subjective rate of intertemporal preferences (the effects of which will be reflected by the savings rate) and the economy openness parameters. As it follows from the general equilibrium model, which does not cover all the factors affecting the equilibrium rate, the long-term equilibrium rate may change as a result of certain shocks, and the above factors are intended to take such changes into account. Regressors can probably be chosen more carefully. Here, we show only the first results for this method based on the statistics that are most available for a long period for a large number of countries.

We use data averaged for the five consecutive periods for 30 developed and developing countries from 1970 to 2014 (nine five-year periods). Such averaging intends to remove the business cycle effect on estimates. Data sources are provided in the Table 1 in Appendix 4. The real interest rate is calculated as the ex-post real rate (very close to the IMF's alternative data). We used the Chinn-Ito index of

²⁸ The central bank's structural policy measures include, for instance, a policy for expanding the planning horizon of economic agents and mechanisms for creating 'long money' supply. In this respect, the inflation targeting policy may be considered a structural policy.

capital account openness (Chinn and Ito, 2006),²⁹ which is measured by its authors on a regular basis, as an indicator of the closeness of dependence on the global economy and financial markets.

To reduce the endogeneity problem, we used the instrumental variables method. The income growth tools that we used included GDP growth at a previous period (we also tried to use the capital account openness indicator at a previous period as a GDP tool), while the share of population aged 15–65 and the current account level (in % to GDP) were used as the savings rate instrument.

The estimates are provided in Appendix 4 (see Table 2). Estimates of coefficients with per capita GDP growth and the savings rate are significant. It follows from the estimates that potential per-capita GDP growth by 1 p.p. causes an increase in the real rate by 1.1 p.p., other conditions being equal, and growth in the savings rate by 1 p.p. corresponds to a reduction in the interest rate by 0.3 p.p.

In the equation (8) we insert figures for Russia, which we expect will characterise the economy's dynamics in the next five years. We estimate the per capita GDP growth in the next five years at 1.9% (the midpoint of the interval of estimates of per capita annual GDP growth mentioned in Section 3.1). The savings rate is set at of 28%, taking into consideration its tendency towards a slow decrease, while the Russian capital account openness index is assumed to stay at the level of 2013. As a result, we obtain an estimate for the equilibrium short-term interest rate at 1.0% per annum. The 95% confidence interval for this estimate turns out to be very wide at $[-10; +12]\%$ per annum.

We also made an estimate of the model based not on the full sample, which includes developed and developing economies, but only on a subsample of developing economies (17 countries). As a result, the sensitivity of the equilibrium rate to the national savings rate increased, and its response to GDP growth became statistically insignificant, even at the 10% level (see Appendix 4, Table 3). Both differences may have a reasonable explanation: they may be attributed to market imperfections and difficulties in smoothing consumption. The mean equilibrium rate level also grew by 0.5 p.p. The overall decrease in the equilibrium interest rate after we inserted the figures for Russia was 0.3 p.p. In our opinion, such instability may be attributed to a different economy structure, which is not captured by explanatory variables. This is a typical criticism for empirical estimates based on panel data.

It is important to emphasise that the insignificance of the statistical relationship between the interest rate and the average GDP growth over five years found in the sample of developing countries is of great practical importance. The insignificance of this relationship means that if there is an observed slowdown in the growth rate of potential GDP in practice (measured as average growth over an extended period), it does not empirically mean that the equilibrium rate in a given country has decreased. In turn, the latter does not mean that it is advisable for the regulator to reduce interest rates to keep monetary policy rigidity at the same level.

²⁹ The index is fixed in its construction and changes in the sample from -1.9 to + 2.4.

5.2. Equilibrium rate estimate in a semi-structural approach: the Laubach and Williams (2003) model

In most research papers, the equilibrium real interest rate is defined as the level of interest rate corresponding to the output dynamics neutral in respect of inflation (this definition is also in line with the natural rate of interest theory by Knut Wicksell (Wicksellian Interest Rate)) and, accordingly, to GDP being at its potential level.

Laubach and Williams (2003) were the first to take a comprehensive model approach to simultaneous measurement of potential GDP, its growth rate and the equilibrium real interest rate for the US economy using the Kalman filter. We reproduced this approach on Russian data. The canonical version of the model proposed by Laubach and Williams (2003) includes three key equations:

- the Phillips curve (a function of inflation expectations and the output gap);³⁰
- the aggregate demand curve (a function of the output gap and the deviation of real interest rate from its natural value in the previous quarter);
- the neutral interest rate equation (dependence on potential GDP and other fundamental factors related to intertemporal preferences).

This approach essentially mimics the loss function of the central bank that chooses the optimum in the context of maintaining low and stable inflation, while at the same time contributing to high and stable rates of economic growth. The approach is supposedly more relevant in terms of estimating the neutral interest rate in the framework of monetary policy, compared to, for example, the general equilibrium model from Section 4.³¹

The model parameters were estimated based on quarterly data for a sample from 2003Q1 through 2015Q2 (50 observations).

The short-term rate on loans to non-financial organisations for a period of up to one year was used as the source indicator of the actual real interest rate. The use of the short-term rate in the interbank credit market in our calculations was to a large extent complicated by the fact that the interbank market rates in the period of high oil prices and chronic liquidity surplus in the banking system before the 2008–2009 crisis were close to the lower bound of the Bank of Russia's interest rate corridor and were, therefore, much lower than the expected inflation. The interest rate corridor itself did not play the role it plays today; the volatility of rates was much higher, and their dynamics were determined by the Central Bank's foreign exchange interventions.

³⁰ During the calculations made using the Laubach and Williams (2003) model with Russian data, we also added export and import price indicators to the Phillips curve equation.

³¹ Nevertheless, since the aim of this paper is to obtain the range of estimates of the neutral rate in the Russian economy in accordance with the established approaches in the literature, and not to issue recommendations for the monetary policy, we do not present the replication of Laubach and Williams approach (Laubach and Williams, 2003) on Russian data as the primary approach. Instead, we view it as yet another one of the possible methods for estimating the equilibrium natural rate.

The actual real interest rate was calculated from the dynamics of the nominal rate and inflation expectations at each moment of time factored in. Retrospectively, an adaptive component prevailed in the inflation expectations of Russian economic agents for objective reasons (the Bank of Russia has been carrying out a monetary policy of inflation targeting only since November 2014). For that reason when calculating the expected inflation we assigned a greater weight of 75% to current inflation and a comparatively lower weight of 25% to future inflation (a component of rational expectations in accordance with ideal foresight). It should be noted that the above-mentioned procedure for calculating inflation expectations had a weak effect on our estimates in terms of the direction of deviations of the actual real interest rate from the equilibrium level.

Figure 9. Actual and equilibrium real interest rate for short-term loans in the economy (Kalman filter estimates for the model with potential output), % per annum

Sources: Bank of Russia, authors' calculations

An important common feature of such estimates of the equilibrium rate is its high uncertainty, for example, for the US, see Beyer and Wieland (2019). For Russia, we have also found a high level of uncertainty in our estimates.

The resulting point estimates indicate an equilibrium rate for short-term corporate loans equal to 3%. According to our estimates, taking into consideration the spread dynamics in respect of the money market rate in that period, this indicates an equilibrium money market rate of about 0.5%. However, this spread is nominal; in other words, it takes inflation volatility premium into account and, therefore, reduces the equilibrium real short rate estimate. Moreover, the width of the 90% confidence interval estimated in the context of replication of the Laubach and Williams (2003) approach is about 4 p.p. Thus, the equilibrium interest rate of the money market specified above with 90% probability is in the interval of $[-1.5; 2.5]\%$. That interval, in turn, is too wide for such empirical estimates to

provide guidance to the decisions on monetary policy in practice, especially when monetary terms are already close to a neutral level.

6. Conclusion

The results of using a large number of approaches to the estimation of the equilibrium (neutral) rate are the following:

1. According to strict definitions, the equilibrium real rate estimates in Russia all have wide confidence intervals.
2. Short-term equilibrium rates are subject to strong and abrupt changes following changes in terms of trade but do not change over the long term. This means that for the set key rate of the central bank, its position relative to the neutral rate may change to the opposite over the short term.
3. Over the long term, equilibrium interest rate estimates in accordance with various partial equilibrium conditions behave differently: interest parity estimates turn out to be lower than the rates from conditions that depend on domestic economic factors. This can be partially attributed to the fact that, unlike interest parity calculated for money market short rates, rates in the other two theoretical conditions are mean ruble rates for deposit and credit operations of agents, households and firms. These rates also factor in both risk premium and term premium and must, therefore, be higher than the money market rates.³² We show that the rate from the equilibrium of consumer choice most closely suits the task of determining the equilibrium rate over the long run in general equilibrium. However, the estimate turns out to be very sensitive to the intertemporal discount coefficient values estimated, which in turn is estimated with wide confidence intervals.
4. Econometric estimates of the equilibrium rate over the long term in the panel data model, linking the money market short interest rate with per capita GDP growth rates and other variables, have wide confidence intervals that are difficult to use in practice when analysing and carrying out monetary policy. Based on the data for 30 developed and developing countries from 1970 to 2014, the current point estimate of the short equilibrium rate over the long term is 1.0% with a 95% confidence interval of [-10; +12]%. The correlation with the growth rate of potential GDP turns out to be statistically significant. Increasing the openness of the economy (especially its capital account) also reduces the equilibrium rate (but the effect is not robust and depends on the specification). In the developing economies, the rate is more

³² The extent depends on the equilibrium excess of the bank's real mean deposit and credit rate over the real short rate of the money market. In 2013–2014, the estimated mean spread of retail deposits and loans with up to one-year maturity was about 7.5 p.p., while for loans/deposits to non-financial organisations with up to one-year maturity the spread amounted to 1.5 p.p. This spread is expressed in nominal values and factors out, for example, the inflation volatility premium, which is hard to measure. This may result in reduced estimates of the real short rate.

sensitive to the savings rate than in the developed ones and is correlated insignificantly with per capita GDP growth. In our opinion, besides possible structural shifts in estimates, this may reflect the imperfection of financial markets that do not allow consumption smoothing.

5. Growth in the global interest rate in the model that we propose in this paper does not affect the long-term interest rate; the latter adjusts to the higher level of the global interest rate in general equilibrium through deleveraging and a decrease in the risk premium. If there are any restrictions for the sensitivity of the risk premium, the type of the model that we consider may have no solution.
6. Estimation on panel data based on semi-structural methods (for the current equilibrium) are also characterised by uncertainty that is too high for practical purposes.

The results we obtained are important both for the formulation of monetary policy and central banks' communication policy.

With respect to conducting monetary policy, our results confirm that the Bank of Russia has to operate in conditions of high uncertainty about the key structural parameters of the economy (this concerns both the real interest rate and the rate of growth in potential GDP). Therefore, developing a robust policy rule that would factor in the dynamics of observable indicators only (inflation, GDP or even oil prices rather than output gaps or rate gaps) becomes especially relevant.³³

In our opinion, amid the uncertainty of neutral interest rate estimates, it is crucial for central banks not only to have and critically think through the results of a wide range of approaches but also to continue estimating the real rates with a focus on their potential application in forecasting macro-indicators as well as on the proper understanding of the response to certain structural changes in the economy. We also see further work we can undertake in examining the optimal choice of the policy by the central bank given the uncertainty of equilibrium rate estimates.

With regard to communication, it is important to clearly specify the applied definition of the rate and its boundaries and restrictions. When communicating the rate estimates, their uncertainty should be indicated, or they should be communicated as a rate forecast without referring to the rate equilibrium value.

In general, it follows from our findings that it is important to avoid using non-observable variables in communication; it is harder for the public to monitor the actions of central banks and for central banks to explain their actions. At the same time, central banks should avoid template rate forecasts or customary practices in communication; each specific case should be addressed with a focus on optimum achievement of the set objectives for stabilisation of the economy.

³³ See Orphanides and Williams (2002).

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APPENDIX 1

Equilibrium (long-term) GDP growth estimates for Russia

We estimate GDP growth using the following formula:

$$g_Y = \frac{g_A}{\alpha} + g_L,$$

where g_Y is the long-term GDP growth, g_A is the long-term growth of total factor productivity (TFP), g_L is the long-term population growth and α is the parameter characterising the share of labour costs in GDP production.

We model the dynamics of total factor productivity as approaching the technological leader (we use the US here) – that is:

$$g_A = g_A^* + k * \left(\frac{Y^*}{Y} - 1 \right) * 100,$$

where g_A^* is the technological leader's TFP growth rate, Y^* is the technological leader's per-capita GDP and Y is Russia's per capita GDP.

For the US, g_A^* is estimated at 2% per annum (Jones, 2016). We use this value as the maximum estimate and take a minimum estimate equal to 1%. The contribution of the approach to the technological leader is assessed at 0.2%–0.6%, depending on the calibration used. It is expected that the working-age population will decrease at a rate of 0%–0.6% per annum.¹ The resultant range of GDP growth estimates is 0.6%–2.6% per annum

Mathematical description

The production function is described by the formula:

$$Y_t = A_t * (L_t)^\alpha * (K_t)^{1-\alpha}.$$

After finding the logarithm and differentiating, we obtain:

$$\frac{dY_t}{Y_t} = \frac{dA_t}{A_t} + \alpha \frac{dL_t}{L_t} + (1 - \alpha) \frac{dK_t}{K_t},$$

or

$$g_{Y,t} = g_{A,t} + \alpha g_{L,t} + (1 - \alpha) g_{K,t}.$$

In equilibrium:

$$g_{K,t} = g_{Y,t},$$

then:

$$g_{Y,t} = \frac{g_{A,t}}{\alpha} + g_{L,t}.$$

¹ Rosstat Demographic Forecast.

Table 1. Growth accounting–based estimates

	Upper limit	Average estimate	Lower limit
Contribution of the technological leader's TFP	2.0%	1.5%	1.0%
Contribution of catch-up development	0.6%	0.4%	0.2%
Contribution of labour	0.0%	-0.3%	-0.6%
GDP growth	2.6%	1.6%	0.6%

APPENDIX 2

RBC general equilibrium model for Russia

Key to the variables used below (the model applies quarterly data):

beta_annual – annual subjective discount factor

B_star – net assets in foreign currency

c – consumption adjusted for technology growth

c_yoy – consumption adjusted for technology growth, year-over-year

d – external debt

h – domestic final goods adjusted for technology growth (in levels)

h_c – domestic final goods included in the investments basket (in levels),
adjusted for technology growth

h_i – domestic final goods included in the consumer basket (in levels),
adjusted for technology growth

i – investments adjusted for technology growth (in levels)

im – imports adjusted for technology growth (in levels)

im_c – imported goods included in the consumer basket (in levels), adjusted
for technology growth

im_i – imported goods included in the investments basket (in levels), adjusted
for technology growth

im_i_yoy – imported goods included in the investments basket, adjusted for
technology growth, year-over-year

i_yoy – investments adjusted for technology growth, year-over-year

k – capital adjusted for technology growth (in levels)

l – hours worked

p_e – relative price of intermediate goods (price of intermediate goods in
terms of consumer goods)

p_f – relative price of imported goods

p_h – relative price of domestic final goods

p_i – relative price of investments

p_oil – relative price of oil in terms of foreign goods

p_x – relative price of non-oil exports in terms of foreign goods
 q – q -Tobin (Lagrange multiplier in the equation measuring capital accumulation)
 q_x – non-oil exports adjusted for technology growth (in levels)
 $q_{x,yoy}$ – non-oil exports adjusted for technology growth, year-over-year
 r_{real} – real interest rate
 rp – risk premium
 R_{star} – foreign interest rate (nominal)
 rer – real exchange rate (increase = weakening of the real exchange rate)
 w – real wages adjusted for technology growth
 x – volume of exports
 y_e – production of intermediate goods adjusted for technology growth (in levels)
 $y_{e,yoy}$ – production of intermediate goods adjusted for technology growth, year-over-year
 zk – real capital value

Figure 1. Diagram of the model

Households

Households choose a level of consumption, savings and hours worked, thus maximising utility, which looks as follows:

$$U_t(j) = E_t \sum_{i=0}^{\infty} \beta^i * e^{z_t^{pref}} * \left(\frac{(C_{t+i}(j) - hC_{t+i-1})^{1-\sigma_c} - 1}{1 - \sigma_c} - A_L \frac{(L_{t+i}(j))^{1+\phi}}{1 + \phi} \right)$$

with budget constraints:

$$P_t^{cpi} C_t(j) + (R_t)^{-1} B_t(j) + \left(R_t^* * rp(b_t^*, p_t^{oil}, e^{RP}) \right)^{-1} \mathcal{E}_t B_t^*(j) = W_t L_t(j) + B_{t-1}(j) + \mathcal{E}_t B_{t-1}^*(j) + \Pi_t(j),$$

where P_t^{cpi} is the consumer price index, R_t is the domestic interest rate, R_t^* is the foreign interest rate, $rp(b_t^*, p_t^{oil}, e^{RP})$ is risk premium depending on the relative debt level (see below) and the real US-dollar-denominated oil price of the stochastic part of the risk premium, $\mathcal{E}_t$ is the nominal exchange rate, W_t is wages, $C_t(j)$ is the consumption of the j -th household, $L_t(j)$ is the labour of the j -th household, $B_t(j)$ is the net national currency assets, $B_t^*(j)$ is the net foreign currency assets and $\Pi_t(j)$ is one-off payments.

The variables are adjusted for the value $F_t = A_t^{(1/(1-\alpha))} \cdot f_t = F_t / F_{t-1}$.

First-order conditions look as follows:

$$1 = \beta * R_t * E_t \left(\frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_c}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_c}} * \frac{P_t^{cpi}}{P_{t+1}^{cpi}} \right), \quad (1)$$

$$1 = \beta * R_t^* * rp(b_t^*, p_t^{oil}, e^{RP}) * E_t \left(\frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_c}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_c}} * \frac{P_t^{cpi}}{P_{t+1}^{cpi}} * \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right), \quad (2)$$

$$\frac{W_t}{P_t^{cpi}} * (C_t - h * C_{t-1})^{-\sigma_c} = A_L * (L_{t+1})^\phi. \quad (3)$$

The labour supply equation used in the model looks somewhat different from equation (3) as we introduce rigidities on the labour market similar to price rigidities.

The aggregator firm aggregates the differentiated labour of households and sells it to firms.

Demand for labour:

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\frac{\phi}{\phi-1}} L_t.$$

Firms

Manufacturers of intermediate goods

The production function of domestic firms depends only on hired labour and the aggregate factor productivity:

$$Y_t^E(k) = A_t * K_t(k)^\alpha * (L_t(k))^{1-\alpha}, \quad (4)$$

where $Y_t^E(k)$ is the production of the k -th firm manufacturing intermediate goods, $L_t(k)$ is the labour hired by the k -th firm, A_t is the aggregate factor productivity, $K_t(k)$ is the capital used by the firm and P_t^E is the established price.

Manufacturers maximise profits:

$$E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \left(\frac{P_{t+i}^E(k)}{P_{t+i}^{cpi}} * Y_{t+i}^E(k) - \frac{W_{t+i}}{P_{t+i}^{cpi}} * L_{t+i}(k) - \frac{ZK_{t+i}}{P_{t+i}^{cpi}} * K_{t+i}(k) \right).$$

Demand for the products of a specific firm (again, on the part of a certain packer)

$$Y_{t+i}^E(k) = \left(\frac{P_{t+i}^E(k)}{P_{t+i}^E} \right)^{-\frac{1}{v_E}} Y_{t+i}^E,$$

where $P_t(k)$ is the price established by the k -th firm, Y_t is the volume of manufactured

household goods and $\lambda_{t,t+i}$ is the discount factor applied by the firm, $\pi_t = \frac{P_t^{cpi}}{P_{t-1}^{cpi}}$.

First-order condition:

$$\left(1 - \frac{1}{v_E} \right) + \frac{1}{v_E} \frac{W_t * L_t}{Y_t^E * (1-\alpha) * P_t^{cpi}} * \frac{1}{p_t^E} = 0, \quad (5)$$

where $p_t^E = \frac{P_t^E}{P_t^{cpi}}$,

$$\frac{W_t * L_t}{ZK_t * K_t} = \frac{1-\alpha^H}{\alpha^H}. \quad (6)$$

Retailer firms

Domestic goods:

$$\max \left\{ E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[\frac{P_{t+i}^H(k) * H_{t+i}(k)}{P_{t+i}^{cpi}} - \frac{P_{t+i}^E * H_{t+i}(k)}{P_{t+i}^{cpi}} - \frac{k_H}{2} * \left(\frac{P_{t+i}^H(k)}{P_{t+i-1}^H(k)} - (\pi_{t+i-1}^H)^{\iota_H} * (\pi_*)^{1-\iota_H} \right)^2 * H_{t+i} * \frac{P_{t+i}^H}{P_{t+i}^{cpi}} \right] \right\},$$

$$H_{t+i}(k) = \left(\frac{P_{t+i}^H(k)}{P_{t+i}^H} \right)^{-\frac{1}{v_H}} H_{t+i}.$$

First-order conditions:

$$\begin{aligned} & \left(1 - \frac{1}{v_H} \right) + \frac{1}{v_H} \frac{p_t^E}{p_t^H} - k_H * (\pi_t^H - (\pi_{t-1}^H)^{\iota_H} * (\pi_*)^{1-\iota_H}) * \pi_t^H + k_H * \beta \\ & * E_t \left(\frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_c}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_c}} * (\pi_{t+1}^H - (\pi_t^H)^{\iota_H} * (\pi_*)^{1-\iota_H}) * \left(\frac{\pi_{t+1}^H}{\pi_{t+1}} \right)^2 \right. \\ & \left. * \pi_{t+1} * \frac{H_{t+1}}{H_t} \right) = 0. \quad (7) \end{aligned}$$

Exported goods (local currency price setting):

$$\max \left\{ E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \left[\frac{P_{t+i}^{*X}(k) * \varepsilon_{t+i} * Q_{t+i}^X(k)}{P_{t+i}^{cpi}} - \frac{P_{t+i}^E * Q_{t+i}^X(k)}{P_{t+i}^{cpi}} - \frac{k_X}{2} \right. \right. \\ \left. \left. * \left(\frac{P_{t+i}^{*X}(k)}{P_{t+i-1}^{*X}(k)} - (\pi_{t+i-1}^{*cpi})^{\iota_X} * (\pi_*)^{1-\iota_X} \right)^2 * Q_{t+i}^X * \frac{P_{t+i}^{*X}(k) * \varepsilon_{t+i}}{P_{t+i}^{cpi}} \right] \right\} \\ Q_{t+i}^X(k) = \left(\frac{P_{t+i}^{*X}(k)}{P_{t+i}^{*X}} \right)^{-\frac{1}{\nu_X}} Y_{t+i}^X.$$

First-order conditions:

$$\left(1 - \frac{1}{\nu_X} \right) + \frac{1}{\nu_X} \frac{1}{rer_t} \frac{p_t^E}{p_t^{*X}} - k_X * \left(\pi_t^{*X} - (\pi_{t-1}^{*X})^{\iota_X} * (\pi_*^{*cpi})^{1-\iota_X} \right) * \pi_t^{*X} \\ + k_H * \beta * E_t \left(\frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_C}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_C}} * \left(\pi_{t+1}^{*X} - (\pi_t^{*X})^{\iota_X} * (\pi_*^{*cpi})^{1-\iota_X} \right) \right. \\ \left. * \left(\frac{\pi_{t+1}^{*X}}{\pi_{t+1}} \right)^2 * \frac{\varepsilon_{t+1}}{\varepsilon_t} * \pi_{t+1} * \frac{Q_{t+1}^X}{Q_t^X} \right) = 0. \quad (8)$$

Importer firms

Importer firms purchase goods abroad and resell them with a markup inside the country, thus maximising profits:

$$E_t \sum_{i=0}^{\infty} \lambda_{t,t+i} \left(\frac{P_{t+i}^F(k)}{P_{t+i}^{cpi}} \left(\frac{P_{t+i}^F(k)}{P_{t+i}^F} \right)^{-\frac{1}{\nu_F}} Im_{t+i} - \varepsilon_{t+i} * \frac{P_{t+i}^{*cpi}}{P_{t+i}^{cpi}} * \left(\frac{P_{t+i}^F(k)}{P_{t+i}^F} \right)^{-\frac{1}{\nu_F}} \right. \\ \left. * Im_{t+i} - \frac{k_F}{2} * \left(\frac{P_{t+i}^F(k)}{P_{t+i-1}^F(k)} - (\pi_{t-1}^F)^{\iota_F} * (\pi_*)^{1-\iota_F} \right) * Im_{t+i} * \frac{P_{t+i}^F}{P_{t+i}^{cpi}} \right),$$

where $P_t^F(k)$ is the price established by the k -th importer firm, Im_t is the volume of imports into the economy, P_t^{*cpi} is the level of prices for foreign goods and

$\pi_t^F = \frac{P_t^F}{P_{t-1}^F}$, $Im_{t+i}(k) = \left(\frac{P_{t+i}^F(k)}{P_{t+i}^F} \right)^{-\frac{1}{\nu_F}} Im_{t+i}$ is the demand for goods imported by an individual firm.

First-order condition:

$$\left(1 - \frac{1}{\nu_F} \right) + \frac{1}{\nu_F} * rer_t * \frac{1}{p_t^F} - k_F * \left(\pi_t^F - (\pi_{t-1}^F)^{\iota_F} * (\pi_*)^{1-\iota_F} \right) * \pi_t^F + k_F \\ * \beta * E_t \left(\frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_C}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_C}} * \left(\pi_{t+1}^F - (\pi_t^F)^{\iota_F} * (\pi_*)^{1-\iota_F} \right) * \left(\frac{\pi_{t+1}^F}{\pi_{t+1}} \right)^2 \right. \\ \left. * \pi_{t+1} * \frac{Im_{t+1}}{Im_t} \right) = 0, \quad (9)$$

where rer_t is the real exchange rate set by the formula $rer_t = \frac{\varepsilon_t * P_t^{*cpi}}{P_t^{cpi}}$.

Investment firms

Investment firms maximise profits:

$$\max_{K,I} E_t \left\{ \sum_{i=0}^{\infty} \lambda_{t,t+i} \frac{ZK_{t+i}K_{t+i} - P_{t+i}^I I_{t+i}}{P_{t+i}^{cpi}} \right\}$$

on the condition:

$$K_{t+1} = (1 - \delta)K_t + e^{\varepsilon_t^I} * \left(1 - \frac{k_I}{2} * \left(\frac{I_t}{I_{t-1}} - f_t \right)^2 \right) I_t. \quad (10)$$

First-order conditions ($q_t = \frac{Q_t}{P_t^{cpi}}; zk_{t+1} = \frac{ZK_{t+1}}{P_{t+1}^{cpi}}; p_t^I = \frac{P_t^I}{P_t^{cpi}}$):

$$q_t = E_t \left\{ \beta \frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_c}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_c}} * (zk_{t+1} + q_{t+1} * (1 - \delta)) \right\}, \quad (11)$$

$$p_t^I = q_t e^{\varepsilon_t^I} * \left[-k_I \left(\frac{I_t}{I_{t-1}} - f_t \right) \frac{I_t}{I_{t-1}} + \left(1 - \frac{k_I}{2} \left(\frac{I_t}{I_{t-1}} - f_t \right)^2 \right) \right] \\ + E_t \left\{ \beta \frac{e^{z_{t+1}^{pref}} (C_{t+1} - h * C_t)^{-\sigma_c}}{e^{z_t^{pref}} (C_t - h * C_{t-1})^{-\sigma_c}} q_{t+1} \varepsilon_{t+1}^I k_I \left(\frac{I_{t+1}}{I_t} - f_t \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}. \quad (12)$$

Balance of trade

The dynamics of net foreign assets are written as follows:

$$\left(R_t^* * rp(b_t^*, p_t^{oil}, e^{RP}) \right)^{-1} * \varepsilon_t * B_t^* = \varepsilon_t * B_{t-1}^* + \varepsilon_t * p_t^{X*} * P_t^{*cpi} \\ * Q_t^X + \varepsilon_t * p_t^{oil} * P_t^{*cpi} * X_t - \varepsilon_t * P_t^{*cpi} * Im_t - \varepsilon_t * P_t^{*cpi} * dRes_t \quad (13)$$

$$d_t = - \frac{\varepsilon_t * B_t^*}{F_t * P_t^{cpi}},$$

where X_t is the volume of exports and $dRes_t$ is the variable responsible for changes in provisions.

Aggregation

$$C_t = \frac{(H_t^{cons})^{\gamma^{cons}} * (Im_t^{cons})^{1-\gamma^{cons}}}{(\gamma^{cons})^{\gamma^{cons}} * (1-\gamma^{cons})^{1-\gamma^{cons}}} \quad (14)$$

$$P_t^H * H_t^{cons} = P_t^{cpi} * C_t \quad (15)$$

$$P_t^F * Im_t^{cons} = P_t^{cpi} * C_t \quad (16)$$

$$I_t = \frac{(H_t^{inv})^{\gamma^{inv}} * (Im_t^{inv})^{1-\gamma^{inv}}}{(\gamma^{inv})^{\gamma^{inv}} * (1-\gamma^{inv})^{1-\gamma^{inv}}} \quad (17)$$

$$P_t^H * H_t^{inv} = P_t^I * I_t \quad (18)$$

$$P_t^H * H_t^{inv} = P_t^I * I_t \quad (19)$$

Market clearing

$$H_t = H_t^{inv} + H_t^{cons} \quad (20)$$

$$Im_t = Im_t^{inv} + Im_t^{cons} \quad (21)$$

$$Y_t^E = H_t + Q_t^X \quad (22)$$

Other:

$$\pi_t^H = \frac{p_t^H}{p_{t-1}^H} * \pi_t, \quad (23)$$

$$\pi_t^F = \frac{p_t^F}{p_{t-1}^F} * \pi_t, \quad (24)$$

$$\pi_t^{X*} = \frac{p_t^{X*}}{p_{t-1}^{X*}} * \pi_t^{F*}, \quad (25)$$

$$rer_t = \frac{\varepsilon_t * p_t^{*cpi}}{p_t^{cpi}} = \varepsilon_t * p_t^{*cpi}. \quad (26)$$

Monetary policy

The Taylor rule:

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*} \right)^{\phi_R} \left(\frac{\pi_t}{\pi_*} \right)^{(1-\phi_R)\phi_\pi} * \exp^{e_t^R}. \quad (27)$$

Real exchange rate targeting has the form:

$$rer_t = e^{e_t^\varepsilon} * rer_{ss,t}. \quad (28)$$

Demand for exports:

$$Q_t^X = e^{z_t^X} (p_t^{X*})^{-\eta_X} Y_t^*. \quad (29)$$

Exogenous processes

$$\log \tilde{z}_t^A = \rho_z * \log \tilde{z}_{t-1}^A + e_t^z$$

$$z_t^{RP} = \rho_{RP} * z_{t-1}^{RP} + e_t^{RP}$$

$$z_t^X = \rho_X * z_{t-1}^X + e_t^X$$

$$z_t^\varepsilon = \rho_\varepsilon * z_{t-1}^\varepsilon + e_t^\varepsilon$$

$$z_t^l = \rho_\varepsilon * z_{t-1}^l + e_t^l$$

$$z_t^{\text{pref}} = \rho_{\text{pref}} * z_{t-1}^{\text{pref}} + e_t^{\text{pref}}$$

$$\log \frac{X_t}{A_t} = \rho_X * \log \frac{X_{t-1}}{A_{t-1}} + e_t^X$$

$$\pi_t^* = \rho_\pi * \pi_{t-1}^* + e_t^{\pi^*}$$

$$y_t^* = \rho_{y^*} * y_{t-1}^* + e_t^{y^*}$$

$$R_t^* = 0$$

$$p_t^{\text{oil}} = 0$$

Additional equations not explicitly included in the model:

$$\log A_t = \log \tilde{g} + \log A_{t-1} + \log z_t^A,$$

$$g = \tilde{g}^{\frac{1}{1-\alpha}},$$

$$z = \frac{z_t^A}{1-\alpha}.$$

APPENDIX 3

Changes in long-term equilibrium

Below are the estimates of the equilibrium levels and impulse responses for all of the three shocks considered in the model. The diagrams represent the response of variables to shocks on a quarter on corresponding quarter of previous year basis (year over year), and start with the values for Q4 as the initial point (the shock takes place in Q1).

Figure 1. Increase of the subjective future discount coefficient by 0.01

Figure 1 continues on p. 42

Continuation, Figure 1 begins on p. 41

Figure 2. Increase of the global interest rate by 1 p.p.

Figure 2 continues on p. 44

Continuation, Figure 2 begins on p. 43

Figure 3. Decrease of the price of oil by 10%

Figure 3 continues on p. 46

Continuation, Figure 3 begins on p. 45

APPENDIX 4

Estimation of the model for the equilibrium rate based on panel data

Table 1. Statistical data (model variables and instrumental variables) for estimation of the model for the equilibrium rate based on panel data

Variable	Indicator	Data source	Unit of measurement
r_{it}	Real interest rate of the money market	IFS IMF Money Market Rates; Consumer Prices. YoY. end of the year; alternative indicator – Real Rates	% per annum
g_{it}	GDP growth per capita in constant prices	World Bank data GDP per capita (constant LCU) YoY	% per annum
s_{it}	Gross savings (% of GDP)	World Bank data Gross domestic savings (% of GDP)	%
$KAopenness_{it}$	Chinn-Ito capital account openness index (2006)	http://web.pdx.edu/~ito/Chinn-Ito_website.htm	coefficient
CA_{it}	Current account of the balance of payments (% of GDP)	World Bank data Current account balance (% of GDP)	%
N_{it}	Share of the population aged 15–64 in total population	World Bank data Population. ages 15–64 (% of total)	%

Countries in the full sample

Argentina. Australia. Brazil. Canada. Chile. China. Hong Kong. Czech Republic. France. Germany. India. Indonesia. Ireland. Italy. Japan. Mexico. the Netherlands. New Zealand. Poland. Russia. Singapore. South Africa. Spain. Sweden. Switzerland. Turkey. USA. Great Britain. Columbia and Peru.

Table 2. Estimation results for model over full sample

Coefficient	Volume
Constant	8.71***
GDP growth per capita	1.15*
Gross savings	-0.35***
Chinn-Ito index	-0.08

Table 3. Estimation results for model over developing countries sample

Coefficient	Volume
Constant	9.28**
GDP growth per capita	1.15
Gross savings	-0.39*
Chinn-Ito index	-0.03

Note: significance levels: *** – 1% level, ** – 5% level, * – 10% level.